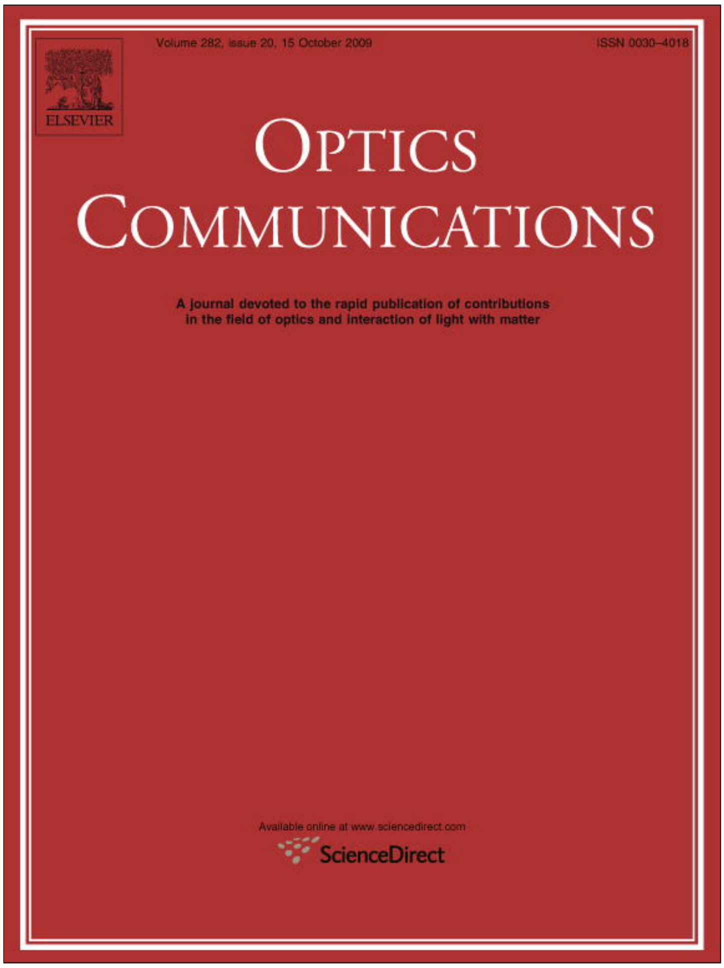
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Two-dimensional temporal coherence coding for super resolved imaging

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ABSTRACT

In this paper, we present an approach that can be used for transmission of 2D spatial information through space-limited systems capable of transmitting even only a single spatial pixel. The input 2D object is illuminated with temporally incoherent illumination. The axial coherence length is very short and it equals only a few microns. Attached to the input object spatial random phase mask generates different axial shift for every pixel of the input. The temporal delays of the encoding (axial shifts) of every pixel are longer than the coherence length of the illuminating source. Therefore no temporal correlation exists between the various pixels of the input. A lens combines all spatial pixels into one point at its focal plane. Although the various spatial pixels were mixed together, since the random mask provided axial delay which was larger than the coherence length of the light source, the orthogonality between the spatial content of every pixel is preserved. The decoding system includes a lens that is positioned at the output of the resolution reduction system and it converts the output light into a plane wave containing all the spatial information of the original image mixed together in all of its pixels. By interfering this plane wave with the same plane wave after passing through the same random spatial coding mask, the spatial information of every pixel of the input object is recovered.

1. Introduction

Super resolution is highly applicable field in optics research where spatial degrees of freedom are encoded transmitted through band limited systems and then decoded while the encoding/decoding process is done by transferring the spatial information into other domains as time [1], polarization [2], field of view [3], gray level [4] and color [5] which are unused according to a priori knowledge that we have on our system [6–8].

One interesting derivative of the time multiplexing super resolving approach is coherence coding. There the spatial information is multiplexed, transmitted through the band limited imaging system and yet later on separated and reconstructed due to the fact that each spatial segment had different fast varying temporal phase of light, i.e. the incoherent mixed segments remain uncorrelated. When coherence of light is used for super resolution two options are possible. The first is related to spatial coherence [9] and the second to temporal coherence [10]. In the first direction the authors have performed shaping of the spatial mutual intensity function of the illuminating beam as a set of orthogonal distributions, each one carrying the information for a different spatial spectral band or spatial region of the input object. In the second approach temporal coherence coding (i.e. each spatial region was added with different axial delay which was more than the coherence length of the light source and that way after the multiplexing the various spatial segments were uncorrelated) was used to code transversal spatial information. Both approaches presented in Refs. [9,10] were demonstrated for 1D objects.

In this paper we expand the operation principle related to coding via temporal coherence but this time for improving the captured resolution of 2D images (that was not described before). Moreover, we push this operation principle to its limit while the 2D image that we use we encode as single spatial degree of freedom. This implies that imaging can be performed even through a pinhole without using an imaging lens at all. In addition, in Ref. [10] the coherence coding was demonstrated for coding and then decoding of two regions in the field of view. This means that we were able to demonstrate field of view multiplexing such that the field of view was effectively increased by a factor of 2. In this paper, not only that we aim to improve resolution rather than to increase the effective field of view but also the improvement (even for 1D objects) is for much larger factors than 2 (the improvement factor is large enough to be able to reconstruct the spatial structure of the object). The proposed approach can be useful in microscopy where usage the proposed approach with objective lens of low NA can yield high resolution in addition to long working distance provided by the low NA of the lens. In addition, it may be applicable for in vitro biomedical imaging where long integration time is feasible.

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In Section 2 we describe the operation principle. Numerical investigation is presented in Section 3. Experimental proof of principle is presented in Section 4. The paper is concluded in Section 5.

2. Technical description

By giving different axial coherence to different spatial pixels of an object, a two-dimensional (2D) image can be folded into a smaller spatial domain (a single pixel, in the limit case) and then be transmitted through the space-limited system. This can be achieved by proper self coherence function (SCF). The SCF is defined as:

$$\Gamma_{11}(\tau) = \langle u(P_1, t+\tau)u^*(P_1, t)\rangle \tag{1}$$

where $u(P, t)$ is an input complex field amplitude, P represents the 2D spatial coordinate, t the time axis, and τ is the time difference between two points. $\langle\ \rangle$ describes ensemble averaging or averaging over time. For an incoherent field $\Gamma_{11}(\tau) = 0$ for all $\tau \neq 0$. In our design, the encoding system is based on the possibility that every spatial region can have an auto-correlation with a unique time delay $\tau(P)$ that will separate this specific spatial region from the others.

The recovery of every region will be based on the time delay that was used for its encoding. The incoherent light that is being used in the encoding system can be described by the temporal phase de-correlation which is obtained after a time that is longer than the coherence time τ_c. The coherence time has a value that is of the same order of magnitude as $1/\Delta\upsilon$, where $\Delta\nu$ is the temporal spectrum bandwidth of the illuminating source. We simulate a broadband spectrum illumination to have short τ_c. Proper encoding will generate a SCF according to:

$$\Gamma_{11}(\tau(P)) = \langle u(P_1, t+\tau(P))u^*(P_1, t)\rangle \tag{2}$$

The decoding process will yield $\Gamma_{11}(\tau(P)) = 0$ for all $\tau(P)$, except when $\tau(P) = \tau(P_1)$, where it has a finite value.

Note that 2D image encoding, transmission through single mode fiber (a system capable of transmitting only a single spatial degree of freedom) and then restoring was demonstrated before [11]. However, there wavelength coding of the 2D image was used and therefore the approach imposes a restriction related to the fact that the image should have uniform reflection of colors within the illuminating bandwidth.

In this paper as previously described we use the temporal coherence of the illuminating source in order to encode the 2D information. Therefore, now the limitations over the properties of allowed 2D objects are significantly reduced. The object can be colored and since the coherence time is very short the object does not have to be too much static as well.

The proposed experimental system that we suggest will contain two sections: the encoding and the decoding. The encoding system will be installed before a low resolution optical system. The decoding system is placed right after the low resolution system and will repair the coded image. The system is described in Fig. 1.

Both, the encoding as well as the decoding systems, are based upon the Mach–Zehnder interferometers. In one of the arms of the interferometer special optical element is placed which produces different temporal delay per each pixels of the 2D image. This encoding element includes different optical paths for different pixels of the 2D field distribution (it is coined encoder in Fig. 1). After the second beam splitter which combines the two arms of the interferometer, we place a lens that couples the 2D field distribution into a system capable of transmitting only a single spatial degree of freedom (such as a single mode fiber or a pinhole). After being transmitted through the pinhole, another collimating lens is positioned at its output to converts the coming light into a plane wave. The coupling lens, the pinhole or the single mode fiber and then the second collimation lens are coined in Fig. 1 as the low resolution system. This plane wave is input to the decoding system which is similar to the encoding one. Once again it is a Mach–Zehnder interferometer with coding spatial element positioned in one of its arms. The element is identical to the element that was used in the encoding system.

Mathematically the field at the input plane is $u(P, t)$ and after the encoding it will become $u(P, t)$+$u(P, t + \tau(P))$ where $\tau(P)$ is an addition of phase done by the encoder (the special optical element). This additional phase which is expressed as time delay depends on the spatial region. After the transmission through the resolution reduction system that is capable of transmitting only a single degree of freedom (such as a pinhole or a single mode fiber) one has an expression for the field and it is proportional to:

$$\iint u(P,t) + u(P, t+\tau(P))dP \tag{3}$$

The spatial averaging dP is since the wave coming out from the resolution reduction system and the collimating lens, is a plane wave with no spatial information.

The decoding system that includes two interferometer arms will have in one arm: $\int u(P,t) + u(P, t+\tau(P))dP$ and in the other $\int u(P, t+\tau(P')) + u(P, t+\tau(P)+\tau(P'))dP$. The total field impinging on the detector equals to:

$$U_t(P') \triangleq \left(\iint u(P, t+\tau(P')) + u(P, t+\tau(P)+\tau(P'))dP\right) + \left(\iint u(P,t) + u(P, t+\tau(P))dP\right) \tag{4}$$

In our case P' is the spatial coordinates vector of the output plane. The intensity in the output plane (after time averaging) equals to:

$$I(P') = \langle U_t(P')U_t^*(P')\rangle \tag{5}$$

Substituting Eq. (4) into Eq. (5) produces 16 terms. Due to the de-correlation and after the averaging the result contains some terms which are averaged to zero, some which are averaged to

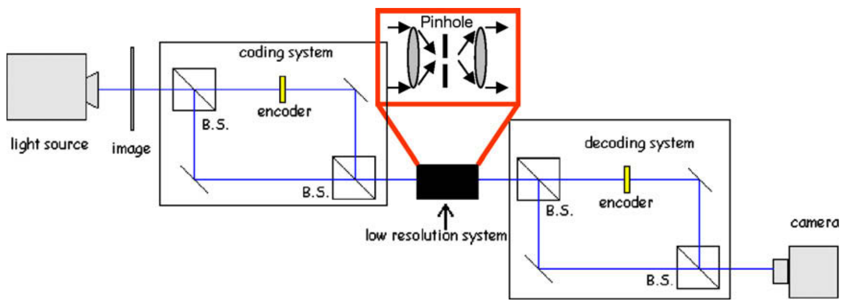

Fig. 1. Optical encoding and decoding system for 2D objects.

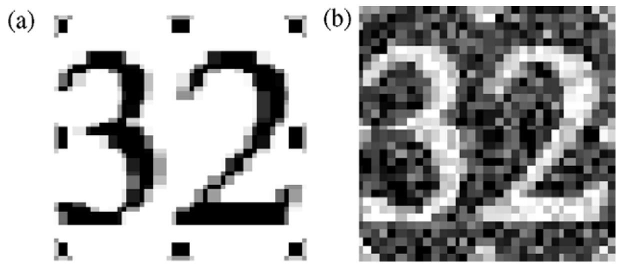

Fig. 2. (a). The original image. (b) Image reconstructed from a single transmitted pixel.

constant of $const = \iint |u(P)|^2 dP$ while the other terms produce some relevant and interesting distribution:

$$
\begin{aligned}
I(P') = {} & const + 2 \cdot Re \\
& \times \left\{ \iiiint \left[\int u(P_1, t+\tau(P_1)) u^*(P_2, t+\tau(P_2)+\tau(P'))dt \right] dP_1 dP_2 \right\} \\
& + 2 \cdot Re \left\{ \iiiint \left[\int u(P_1, t+\tau(P')) u^*(P_2, t+\tau(P_2)+\tau(P'))dt \right] dP_1 dP_2 \right\} \\
& + 2 \cdot Re \left\{ \iiiint \left[\int u(P_1, t+\tau(P_1)) u^*(P_2, t+\tau(P'))dt \right] dP_1 dP_2 \right\} \qquad (6)
\end{aligned}
$$

Note that the temporal average yields:

$$
\begin{aligned}
& \int u(P_1, t+\tau(P_1)) u^*(P_2, t+\tau(P_2)+\tau(P'))dt \\
& \quad = u(P_1) u^*(P_2) \delta(\tau(P_2) + \tau(P') - \tau(P_1)) \\
& \int u(P_1, t+\tau(P')) u^*(P_2, t+\tau(P_2)+\tau(P'))dt \\
& \quad = u(P_1) u^*(P_2) \delta(\tau(P_2)) \\
& \int u(P_1, t+\tau(P_1)) u^*(P_2, t+\tau(P'))dt \\
& \quad = u(P_1) u^*(P_2) \delta(\tau(P') - \tau(P_1)) \qquad (7)
\end{aligned}
$$

assuming for a minute that the relative delay $\tau(P)$ is proportional to its coordinate P: $\tau(P) = \text{constant} \times P$. This happens if for instance the encoder is a prism. In this case the intensity of the output plane becomes:

$$
\begin{aligned}
I(P') = {} & const + 2 \cdot Re \left\{ \int u(P_2 + P') u^*(P_2) dP_2 \right\} + 2 \\
& \cdot Re \left\{ u^*(0) \cdot \left[\int u(P_1) dP_1 \right] \right\} + 2 \cdot Re \left\{ u(P') \cdot \left[\int u^*(P_2) dP_2 \right] \right\} \qquad (8)
\end{aligned}
$$

The second term is an auto-correlation expression that generates strong peak in the center of the axes. The third term is constant and the last term is the one which is the most relevant since it is proportional to the original high resolution field.

Note that in order to obtain this we assumed that $\tau(P)$ is proportional to its coordinate P which is true, as mentioned before, in the case of a prism used as the encoder element. However, in case that the encoder is a random element (as we will assume in our numerical investigation), then the second term does not produce a delta (auto-correlation) but rather it is equal to zero since in the random case the equation of

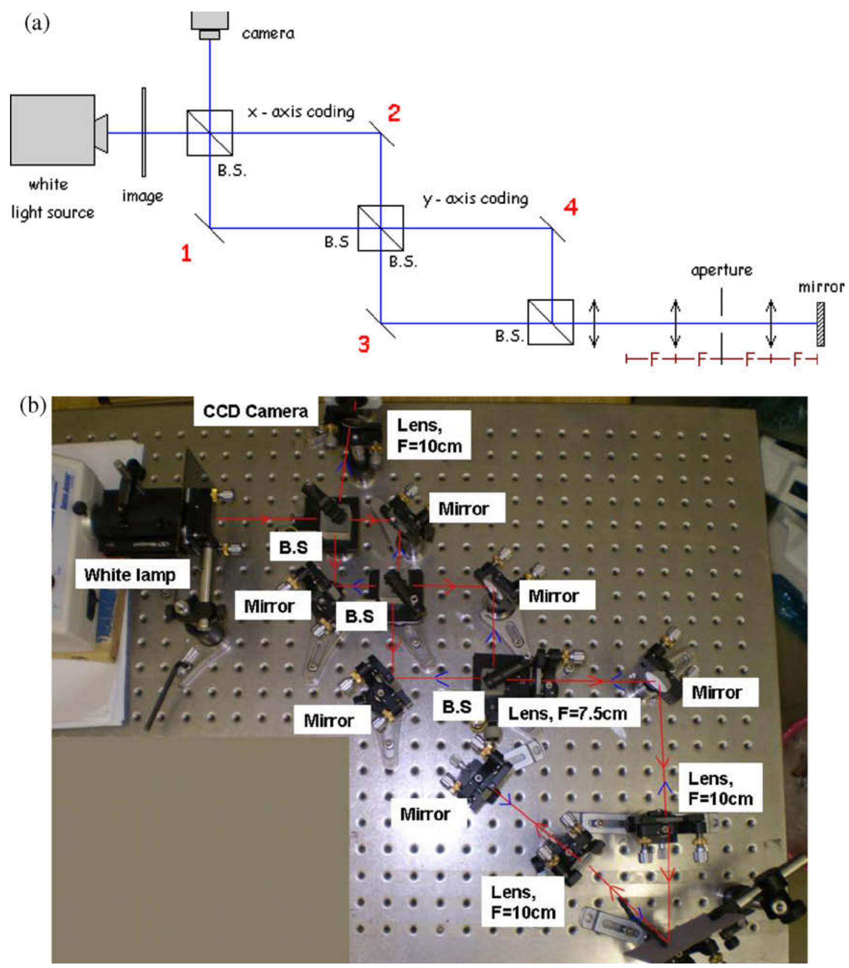

Fig. 3. The realized experimental setup. (a) The schematic sketch. (b) The actually experimental setup with all the optical parameters marked on top.

$$\tau(P_2) + \tau(P') - \tau(P_1) = 0 \quad (9)$$

which is the argument of the delta from Eq. (7), has no defined solution.

Note also that in order to have 2D super resolved imaging, every pixel of the object should be encoded by different optical path such that no correlation will be generated between the pixels after their spatial mixing. Encoding element having the structure of a prism can not work for 2D case but only for 1D case because in a prism there are always pixels having the same optical path. The different optical paths are generated only in 1D. In the case of a random encoding element 2D super resolution can be obtained while every pixel has different optical path and the difference should be more than the coherence length in order to break the correlation between the various pixels.

In any case the interesting result is the last term of Eq. (8) which shows mathematically that indeed the averaged intensity is proportional to the high resolution field distribution of the input object, despite the fact that it was passed through a system that is capable of transmitting only a single degree of freedom. In a sense it resembles what happens in regular holography where the interesting interference term (the coefficient of the fringes), in the intensity distribution of the reconstruction pattern (obtained after the hologram is being illuminated by the reference beam) is proportional to the field of the recorded object.

3. Numerical experimenting

In order to exam the system we have developed a computer simulation that is analogous to the system that consists of five components. The first component is white light illumination source that is synthesized as a matrix with amplitude of one and a random phase. The second part is the image that is being illuminated by the white light source. The third part is the encoding which is achieved by passing the field distribution through the Mach–Zehnder interferometer while one arm passes as is and the second arm is multiplied by a random phase matrix which is the encoding element (coined the encoder). In the encoder matrix each pixel of the 2D image gets a different time delay. At the output of the encoding system the two arms are combined. The next stage is the spatial compression, i.e. the transmission through the resolution reduction module. This is achieved by spatially summing all the pixels into a single term, i.e. spatial integration. The image is compressed into a single spot, a single pixel, and then is expanded into a plane wave (i.e. the single value is replicated for all the pixels of the spatial matrix). Optically this is obtained using the collimation lens. Now the light is input into the decoding system. It passes through additional Mach–Zehnder interferometer, i.e. the light is split into two arms one is multiplied by the encoder matrix (the same filter as in the encoding system) and the other passes as is. At the output both arms are combined.

The phase of the input source is randomly varied with time due to its lack of temporal coherence. Note that the encoder element includes etching depths which are larger than the coherence length (it can be only a few microns) and therefore the phase generated in this path is uncorrelated with the phase in the other path of the interferometer. For every temporal variation of phases the phases of the two arms of the interferometer are varied differently due to this lack of correlation. After sufficient time integration (each captured image is for different initial random phase of the light source, and the encoders) in the detector we reconstruct the original high resolution 2D image.

The obtained result is presented in Fig. 2. In this simulation we used image of 32 by 32 pixels and we performed averaging of 50×10^6 time frames to obtain the final reconstruction. Note that such an averaging occurs after 83 ns for light at frequency of 6×10^{14} Hz (wavelength of 0.5 μm). Another important parameter for the simulation is the coherence length. In our case we used a white light illumination, i.e. the coherence length was approximately equal to one wavelength (half a micron in our case). In Fig. 2a we present the original 2D image which is the number 32. After the reconstruction, we obtain the image of Fig. 2b. One may see that despite the fact that the 2D spatial information was passed through a resolution reduction system (capable of transmitting only a single degree of freedom) the information was fully recovered as can be expected accordingly to the SCF theory and the mathematical analysis presented in the previous section.

4. Experimental proof of principle

Experimental setup following the schematic configuration of Fig. 1, was constructed. The setup is presented in Fig. 3a and b where Fig. 3a is the schematic sketch and Fig. 3b is the real image of the experimental setup with all the optical parameters, such as the focal lengths of the lenses, marked on top. Instead of encoding and then decoding modules we placed a mirror at the output of the encoding module such that the back reflected light will pass through the same system that will now decode and reconstruct the spatial information. The aperture that reduced the imaging resolution of the system had diameter of 1 mm.

The illumination used in the experiment was a white light Halogen lamp having coherence length of about half a micron. The

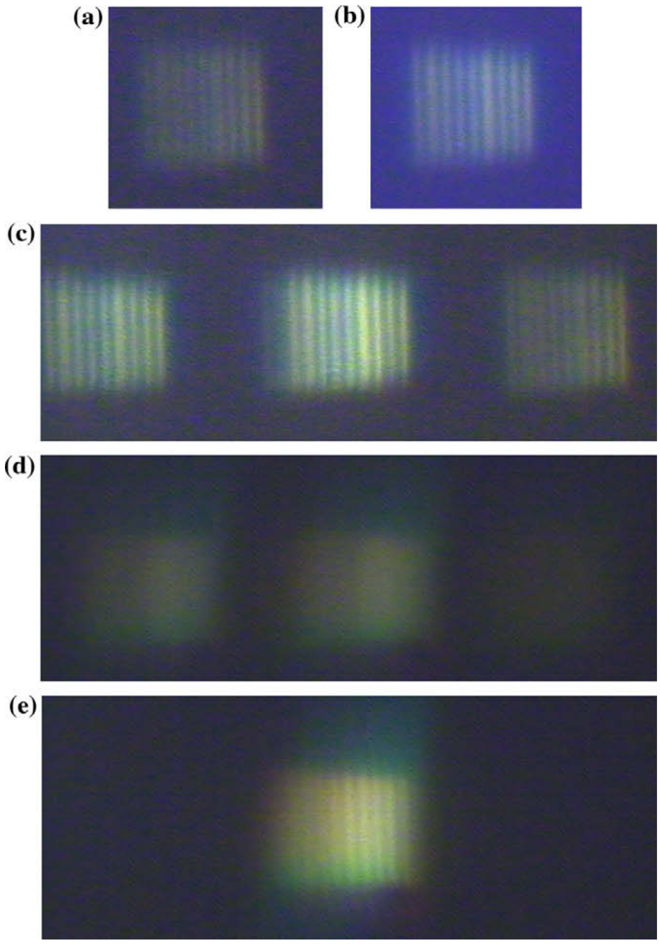

Fig. 4. 1D experimental results. (a) Path 1 was blocked and light went through the path of 2–3. (b) Path 2 was blocked and light went through the path of 1–3. (c) Paths 1 and 2 are open and only path 4 is blocked. (d) The low resolution image obtained due to the insertion of the resolution limiting aperture. (e) The super resolved reconstruction.

light source module is called fiber-lite MI-150 and it is made by Dolan Jenner. The encoding/decoding element was basically two prisms: one for the X encoding and one for Y encoding. Because of the difference in the optical paths between the X encoding module and the Y encoding module (which was much more than the coherence length) the two encodings were uncorrelated and thus a full 2D super resolved imaging could be obtained (rather than only a "plus" like shape synthetic aperture in case that the X and the Y axes are correlated). The tilting angle of each prism determines the final resolving capability. Since the coherence length is about half a micron, the lateral spatial resolution that may be obtained will be half a micron divided by the tilting angle of the prism. This is a required condition in order to generate de-correlation between the various lateral pixels that are being encoded/decoded via the coherence of the light.

The obtained results for the 1D case are presented in Fig. 4. In Fig. 4a we see the image captured when path 1 was blocked and light went through the path of 2–3. Fig. 4b is the image was captured when path 2 was blocked and light went through the path of 1–3. The image of Fig. 4c is captured when paths 1 and 2 are open and only path 4 is blocked. The low resolution image obtained due to the insertion of the resolution limiting aperture is seen in Fig. 4d. The obtained reconstruction is seen in Fig. 4e where the angles of path 1 and 2 were reduced such that the replications were overlapped one on top of the other. In Fig. 4e one may well see the high resolution reconstruction obtained for the spatial information that did not pass through the aperture when the approach was not applied (as seen in Fig. 4d).

In Fig. 5 we performed the experiment for the 2D case. In Fig. 5a one may see the USAF resolution target as it is imaged through our low resolution (due to the 1 mm aperture appearing in Fig. 3) imaging system. For comparison in Fig. 5b we present the high resolution image captured without the aperture. In Fig. 5c and 5d we present the super resolved output obtained after super resolving only the horizontal and then only the vertical axes respectively. One may clearly see the obtained 1D improvement

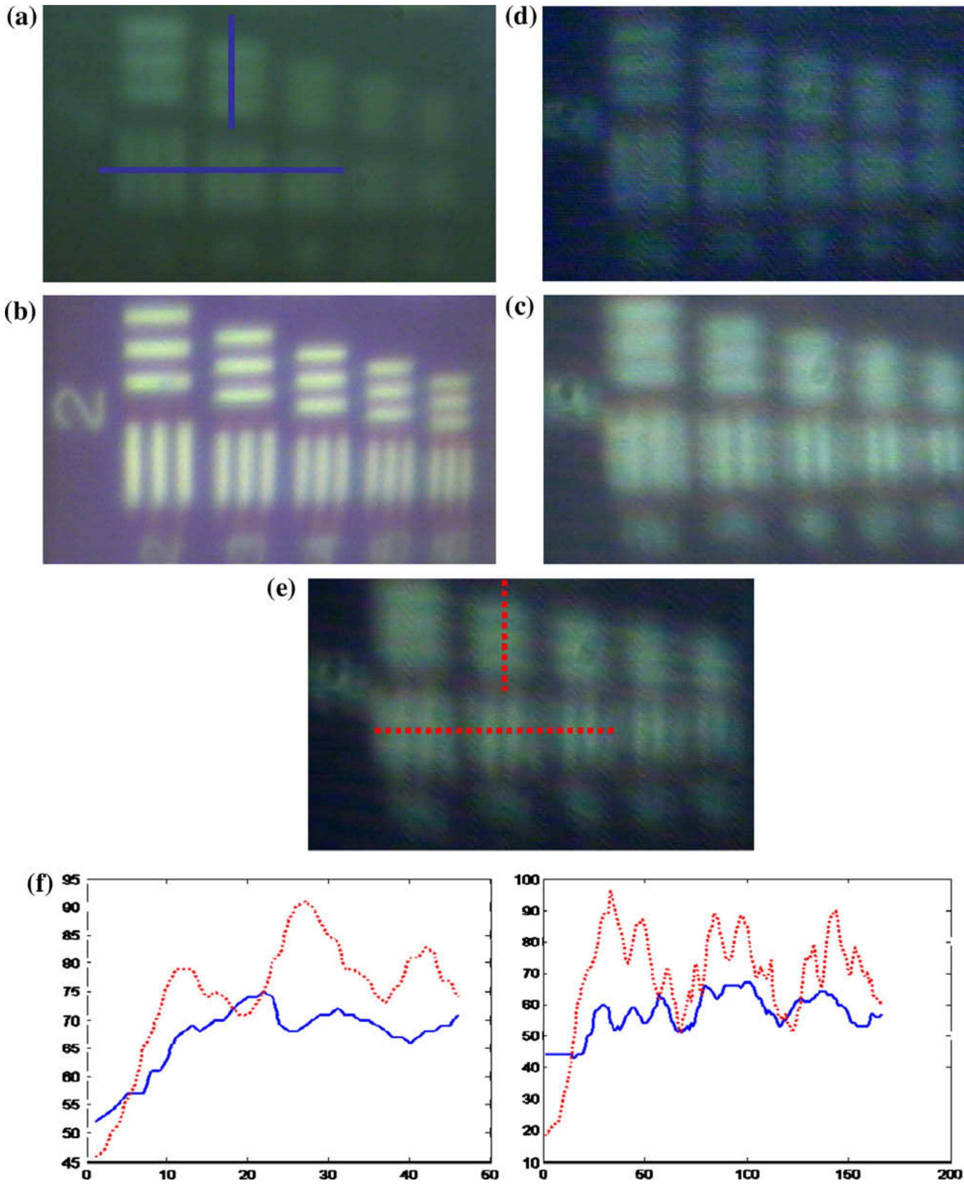

Fig. 5. 2D experimental results. (a) Low resolution image taken with the aperture. (b) The high resolution image taken with the aperture removed. (c) The super resolved image after performing super resolution only in the horizontal axis. (d) The super resolved image after performing super resolution only in the vertical axis. (e) The 2D super resolved reconstruction. (f) Vertical (left) and horizontal (right) cross sections of (a) (solid blue) and (e) (dashed red). (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

(once horizontal in Fig. 5c and then vertical in Fig. 5d) in the imaging capability of high spatial frequencies. In Fig. 5e we present the 2D super resolved reconstruction. One may clearly see the appearance of the high spatial frequencies in both directions (horizontal and vertical) that previously were completely not resolved in the low resolution image of Fig. 5a. Note that the alignment artifacts obtained in the reconstruction of Fig. 5e are caused mainly due to the limited accuracy that we had for the various stages used to align the mirrors. Due to this limited accuracy the four images (two horizontal and two vertical) could not be perfectly recombined.

For the sake of comparison in Fig. 5f we present the vertical (left) and the horizontal (right) cross sections of Fig. 5a (solid blue) and Fig. 5(e) (dashed red). One may see that although some alignment related artifacts appear in the reconstruction of Fig. 5e and thus also in the relevant cross sections of Fig. 5f, nevertheless in the vertical cross section the dashed reconstruction has clearly visible three peaks which are not resolved at all in the solid curve corresponding to the original image of Fig. 5a. In the horizontal cross section one may see that in the dashed curves there are three groups of reconstructed peaks while in the solid curve (corresponding to the image of Fig. 5a) the peaks are resolved only in the first group.

Note that the cross sections presented in Fig. 5f are marked in the relevant positions of Fig. 5a and e.

5. Conclusions

In this paper we have demonstrated the usage of temporal coherence in order to perform 2D super resolution where 2D image is encoded, transmitted through a resolution reduction system and then reconstructed at the output plane. Despite of the demonstrated example, the proposed approach can be used as 2D super resolving approach for any system with limited resolution that can transmit even only a single spatial pixel.

The main advantage of using the temporal coherence as a way to code spatial resolution is related to the fact that although this approach requires time integration, the integrating window can be very short due to the fast variations of the random phases of the source (it can be as fast as the optical frequency). Therefore this does not impose any real restriction and this approach can be used for resolving and imaging of non static objects as well.

The proposed approach can be a useful tool in microscopy where usage of the proposed approach with objective lens of low NA can yield high resolution in addition to long working distance provided by the low NA of the lens.

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